How to solve the Rubik's Cube

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This page describes one of many methods of solving the popular Rubik's Cube puzzle.

Notation

In this article there are several notations, please refer to this page here (http://www.vanderblonk.com/cube/yy/#section3)

Briefly:

- There are six sides to the cube presented as Front, Back, Left, Right, Up and Down, and are usually referred to by their one-letter abbreviations
- In the isometric diagrams below, where a corner points out at you, you see the F, R and U sides. The F faces to the left
- Movements are presented as one quarter rotation (90-degree) of an external face per movement. This means that the center tile colors are not changed. In our diagrams, F is blue, R is red and U is yellow. The other three colors are typically orange opposite red, green opposite blue and white opposite yellow
- Quarter-rotations of that face's layer default to clockwise. Counter-clockwise rotations are often referred to as "inverted" and indicated by an apostrophe (AKA prime, tick mark, anti-clockwise, anti or "i" for inverted). Half-rotations (180 degree) are indicated by the digit "2," meaning 2 quarter-rotations following the one-letter abbreviation
- In order to display what is happening on the sides of the other three colors, the cube will be rotated as a whole described as rotating along the x,y,z space axis, all pointing out of the page. x is R, y is U and z is F, but since this sort of move also changes the colors of the center-tiles, it is used sparingly

Example solve

As an example let's consider a complete solve. 25 move scrambles are used to mix up the cube. Our sample scramble is:
UB'R2D'U'RU2BR'B2L2RF2R2U2RBU2F2L2F2DRB2R2 - demo: scramble (http://vanderblonk.com /cube/cubeapplet.asp?alg=UB'R2D'U'RU2BR'B2L2RF2R2U2RBU2F2L2F2DRB2R2&scripttype=Generator)

The solve is
R'BRD2F2LUFUR'DRFDF'FD'FU2R'D'RU2UF'D'FKKUUB'D'BU'y2FD2F2RF'R'DFD'BDF'BDFF2DMD2M'DF2 (54 moves) - demo (http://vanderblonk.com /cube/cubeapplet.asp?initscript=UB'R2D'U'RU2BR'B2L2RF2R2U2RBU2F2L2F2DRB2R2&alg=R'BRD2F2LUFUR'DRFDF'FD'FU2R'D'RU2UF'D'FUUB'D'BU'y2FD2F2RF'R'DFD'BDF'BDFF2DMD2M'DF2)

Step 1: Top edge pieces (cross)
Remember that the Rubik's cube is very difficult to solve and it requires a lot of patience. So let's get started!

The cube is assumed to be scrambled like crazy. The first thing to do is to choose a color, say white (it tends to stand out from the other colors on the cube). It's also a very good idea to always do a specific color first, since you will remember which colors are adjacent, which speeds things up considerably.

The first step is to form a cross on the top face of the cube. Orient the cube so that the white center piece is on top. The aim is to get the correct pieces in the UL, UB, UR and UF locations. If you started with white these pieces will be colored white-red, white-orange, white-blue and white-green (using a Rubik's brand cube). So, some of the following moves are needed: (be sure to do those in the first step first).

- If a white-other color edge piece is on the U face:
  - If white is in the U position, simply rotate the U face until the edge is lined up with its center. You should now see four stickers connected, e.g. white (center), white (edge), red (edge), red (center).
  - If the piece is flipped in the up layer, then place the edge in the front layer and perform F U' R U [demo (http://vanderblonk.com/cube/cubeapplet.asp?initscript=z2&alg=FU%27RU&stickers=cross)].

- If the piece is in the middle slice of the cube (the second layer):
  - Hold the cube so that white is still on the U face and your piece is in the FR location. Find the spot where the piece should go. Rotate U until either F' or R can be applied to move the piece in the correct spot, so that the white face will move to the top. Make sure that the order of the edges in the top layer will still be correct. On a Rubik's brand cube the order would be blue, red, green, orange. After placing the piece in the top layer, you can turn the U face until all correctly placed edges connect to their centers. [demo F (http://vanderblonk.com/cube/cubeapplet.asp?initscript=z2&alg=UF%27&stickers=cross)][demo R (http://vanderblonk.com/cube/cubeapplet.asp?initscript=z2&alg=UR&stickers=cross)]. An alternative to this method is to move the white sticker to D face (if possible without disturbing the current cross pieces) and move on to the next step.
  - If the white is on the D face, simply rotate D (or D') until the piece is directly underneath its center, and apply F2 (assuming the piece is at the FD position) to put it in the correct location. [demo (http://vanderblonk.com/cube/cubeapplet.asp?initscript=z2&alg=D%27F2&stickers=cross)]
  - If the other color is on the D face (the flipped version of the previous state), keep the piece in the F layer. Rotate D so that the piece is in the RD position, and apply R F' R' [demo (http://vanderblonk.com/cube/cubeapplet.asp?initscript=z2&alg=RF%27R%27&stickers=cross)]. (R' is not needed if the UR piece has not been placed correctly yet).

There should now be a white cross formed on the top of the cube. By now, it will be possible to think how the edge pieces are located relative to one another, which should speed things up.

**Step 2: Top corner pieces**
The second step is to correctly position three of the U face corner pieces. The reason that only three of them, and not four, will be put into place is that this method uses a "working space" which greatly simplifies the later steps.

There are three basic possibilities for putting corner pieces into place:

- The piece is on the D slice, with the white side not on the D side. In this case, rotate the D face so that it is directly underneath the location that it should go to. Now, hold the cube so that the piece is in the DRF spot, and the intended location is the UFR spot.
  - If white is on the R side of the corner piece in DRF, apply R' D' R. [demo](http://vanderblonk.com/cube/cubeapplet.asp?initscript=z2&stickers=FL&alg=R'D'R)
  - If white is on the F side of the corner piece in DRF, apply F D F'. [demo](http://vanderblonk.com/cube/cubeapplet.asp?initscript=z2&stickers=FL&alg=FDF')

- The corner piece is on the D slice, but the white face is on the D side. Rotate the D face so that the corner piece is in the DRF spot, and the intended location is the URF spot. Now apply R' D2 R D R' D' R. [demo](http://vanderblonk.com/cube/cubeapplet.asp?initscript=z2&stickers=FL&alg=R'D2RDR'D'R)

  Note that what is being done is R' D2 R to move the white side off the bottom of the cube, so that one of the moves in the previous section can be used. Also note that equivalent to this is: F D2 F' D' F D F'. [demo](http://vanderblonk.com/cube/cubeapplet.asp?initscript=z2&stickers=FL&alg=FD2F'D'FDF')

For left-handed people this might make things easier. A right-handed person would probably do the "R' D2 R..." move, without thinking about it.

- The corner piece in question is in the right spot but incorrectly rotated. Therefore, it must be rotated. Hold the cube so that it is in the UR'F location. Now,
  - If the white side is on the R face, apply R' D' R D R'. [demo](http://vanderblonk.com/cube/cubeapplet.asp?initscript=z2&stickers=FL&alg=R'D'RDR'D'R)
  - If the white side is on the F face, apply F D F' D' F D F'. [demo](http://vanderblonk.com/cube/cubeapplet.asp?initscript=z2&stickers=FL&alg=FDF'D'FDF')

So now one side should be done, except for one corner piece. This location will be used to swap corner pieces in and out, greatly simplifying later processes. The moves in the first two steps are really quite intuitive. After only a few repetitions, they will be simple and natural to do.

**Step 3: Middle edge pieces**

This step involves correctly placing three of the four edge pieces on the "middle" layer of the cube. For these moves hold the cube so that the white face is on the bottom. The only middle layer edge piece that is not to be positioned is the one right above the corner piece that was not positioned correctly in step 2.

Before you begin positioning edge pieces, you should rotate the middle layer so that all of the center pieces are correctly positioned.

First of all, make sure the white side is on the bottom, and the "empty" (i.e., incorrect) corner piece on the white side is in the DRF location. The middle layer edge pieces will all be positioned in this step, except for the FR one.

To move a piece into position, rotate the cube about its vertical axis, so that the intended location is the FR location. (For example, the FL piece is to be put in place. Rotate the cube a quarter turn counter-clockwise.) Now rotate the bottom slice so that the incorrect corner piece is in the DRF location. (So in the previous example—for the FL piece—first turn the cube, then apply D'.) Now all is prepared for the move. The move to put the new edge piece into place can only be done if it is on the U slice. If it is, note which side is NOT on the U face. Either F' or R will need to be applied,
depending on the orientation of the edge piece to be moved. Now, apply U until the piece to be moved is in the UF or UR (depending on the previous move) location, and then F or R', to get it back to normal.

Here is an example: Yellow is the F center. Orange is the R center. The Yellow-Orange edge piece is to be positioned, to the FR position. The D face has already been rotated so that the DRF location does not contain a white corner piece. The Yellow-Orange piece is in the UB location. The Orange is the U side, and Yellow is the B side. Thus, apply F' U2 F.

[ [demo](http://vanderblonk.com/cube/cubeapplet.asp?stickers=F2L&alg=F'U2F) ]

To continue, simply keep rotating D or D' and moving the cube to set up the same position, with an "empty" corner in DRF, the intended location at RF, and the piece to move in the U slice. Note that in some cases the piece may already be in the correct location, but oriented incorrectly. In this case take it out first (i.e. put any edge piece with the color whose center is opposite white on the cube into that location) and then put it back in that spot. In other words, with the DRF corner "empty" and the offending piece in the FR spot, apply F' U' F U R' R'.

[ [demo](http://vanderblonk.com/cube/cubeapplet.asp?stickers=F2L&alg=F'U'FRU'R') ]

Now two thirds of the cube should be done, less two pieces: a middle layer edge piece and its adjacent corner piece, that appears to take a chunk out of the bottom (white) layer. Note that it is possible for the "empty" corner piece on the bottom layer to get solved by accident. If so, just ignore it, and pretend that it is unsolved.

**Step 4: Solve remaining edge pieces**

**Solve first three remaining edge pieces (UF, UL, UB)**

This is the only step that requires any actual memorization. The moves from the other steps should become very natural after a short time. There are two basic parts to this step, as follows: The goal of the whole step is to solve all of the 5 remaining edge pieces. The first part is to solve three of these (UF, UL, UB), and the second part is to solve the other two together.

First of all, hold the cube so that the "empty" edge piece is in the BR position, and thus the "empty" corner piece is in the RDB position. To do moves in this part, first of all move a piece into the BR location, then move it to the right position, to one of those UF, UL, or UB positions. The move is as follows. First, optionally rotate U. Then, apply R’ or B. Then rotate U the desired amount. Then do R or B' (to undo the first part of this move).

An example: Say the Blue-Yellow piece is in the BR location. Furthermore, Blue is the U color, and Yellow is the L color. The order would be: U [to put the UL location (the destination) in the right spot] B U' B'. However, when actually trying to solve the cube quickly, before applying U in the previous move, look to find the next edge piece that is required to put in the right location. So rotate U until it is in the UB location, and then by applying B' return the cube to a stable position. Then, rotate U some amount to get the UL piece (Blue-Yellow in the example) back to the right place. There is a tremendous amount of freedom in this sequence of moves. In fact, there is no need to return the edge pieces to the correct spots in between repetitions of this move. Simply realize how the pieces go with respect to one another, and then finally align them, when all three (UF, UL, UB) are done.

**Remaining two edge pieces (BR, UR)**

Now, there are four possibilities. The remaining edge pieces are the BR piece and the UR piece. Do the following:

- Lucky, the pieces are correct. Move to the next step.
- The pieces are in the correct locations, but incorrectly oriented. Apply B U' B' U R' U R U'.
- Both edge pieces (BR and UR) have the same color on the R side of the piece, which is the same color as the R center. Apply U' R' U R' U' R U'. [demo](http://vanderblonk.com/cube/cubeapplet.asp?stickers=F2L&alg=U'R'UR'UR'UR')
- The other case (the UR piece has the R color on its U side, and B color on its R side, and the BR piece has the U color on its R side, and the R color on its B side). Apply B U B' U B U' B' U2.

To reduce memorization at the expense of some speed, two of these moves suffice. In other words, apply all three of these moves in any sequence to an all-edges correct cube, and the result will be an all-edges correct cube.

**Step 5: Position corner pieces**

This step will move the remaining 4 unsolved corner pieces to their correct positions, irrespective of orientation. The basic strategy is to move the "empty" corner piece to DRB and the corner piece to be moved to UFL. A move (L D2 L') is used to swap the two corners (as well as temporarily jumble up the cube). Then U slice is rotated such that the correct position for the corner (now in DRB) is in UFL. Then the move (L D2 L') is applied again to undo the cube jumbling as well as move the corner to the correct position.

This shouldn't need to be done more than 3 times (since there are only 4 corners to move and the last two will swap with each other to their correct positions).

This step can be a little confusing at first. First of all, make sure the DRB piece is that "empty" (unsolved...not missing!) corner piece. Say the UFL piece is Blue-Yellow-Orange. But that piece should go in the URB position. Do the following moves: L D2 L' [move the piece in question out of the way (to the DRB position)] U2 [move the correct position to the UFL spot] L D2 L' [move the piece in question back to the U slice] U2 [undo the U twist done earlier]. One thing to note when doing this move, make sure the original UFL piece does not contain the color of the bottom face (white in the ongoing example). Also note that it is satisfactory to rotate the U face before the move so that a particular corner piece can be moved into the UFL position so that it can be worked on. The only (slight) difference will be a need to rotate U at the end to make up for that. Note that these U-rotations should be very obvious. Simply line up the top-layer edge pieces with their respective centers.

**Step 6: Orient corner pieces correctly**

Corner pieces must be rotated in pairs—one clockwise and one counter-clockwise. If you combine two clockwise or counter-clockwise rotations, the rest of your cube will be compromised.

Find two incorrectly rotated corner pieces that are on the same slice. Hold the cube so that one of the pieces in the UFL position and the other is somewhere on the U slice.

- To rotate a piece clockwise, apply L D2 L' F' D2 F. [demo](http://vanderblonk.com/cube/cubeapplet.asp?stickers=LL&alg=LD2L'F'D2F)
- To rotate a piece counter-clockwise, apply F' D2 F L D2 L'. [demo](http://vanderblonk.com/cube/cubeapplet.asp?stickers=LL&alg=F'D2FLD2L')

Note that after orienting the first corner piece, apply U until the other corner piece goes in the UFL location. When the second corner piece has been oriented, turn U to undo the previous twisting (this should be fairly obvious). Here's an explicit example—the UFL piece needs rotating counter-clockwise, and the UFR piece needs rotating clockwise. The full sequence would be as follows: F' D2 F L D2 L' [orient UFL piece] U [position other corner] L D2 L' F' D2 F [orient original UFR piece] U' [undoes rotation of U that was done earlier]. [demo](http://vanderblonk.com/cube/cubeapplet.asp?stickers=LL&alg=F'D2FLD2L'ULD2L'FD2FU')

This pattern may need to be applied up to three times. Note that with this method only one clockwise and one counter-clockwise twist can be done; other methods twist 3 corners but have side-effects on edges. If the two remaining corner pieces are diametrically opposed (e.g. at UFL and DRB), then apply R2 (in this case) to bring both of them onto the U slice. Then, do the sequence. Then apply R2 again to get to the original configuration. [demo](http://vanderblonk.com/cube/cubeapplet.asp?alg=R2F'D2FLD2L'ULD2L'FD2FU'R2)

Congratulations, your cube should now be solved!!
Rotating the center faces

Some cubes have multi-color designs on each face rather than a single color, in which case the orientation of the center faces is an issue. Usually, solving the cube with the center faces oriented correctly is possible, and the fastest method; but there exists a (very slow) way to rotate two of the center faces at a time without affecting the rest of the cube. If you want to rotate the F and R center faces (both clockwise), simply repeat F R a full 105 times after each other. If you want to rotate F clockwise and R counter-clockwise, do F R', etc.. This method can be very time-consuming, however, so planning ahead while solving the rest of the cube is preferred - that is, match the side center faces to the top edges, and hope the bottom turns out correctly. This less time-consuming sequence will rotate the top center face counter-clockwise (ccw) and the left center face clockwise (cw), each $n$ quarter turns: $(L'R'F'B'U'D')^n$. With different notation, the sequence ($n=1$) can be abbreviated $(M E M')^n (D'U B'F R'L)^n$. With this method you must learn around 70 algorithms. There are methods just as fast that require far fewer algorithms to be memorized. Here is a brief synopsis of several popular speedcubing methods:

Layer by Layer methods

Fridrich Method: A very fast First 2 Layers (or F2L) method, start by solving a cross on one face, then proceeding to solve the First 2 Layers pairing up edge and corner combinations and putting them into their slot. This is followed by solving the Last Layer in two steps, first orienting all pieces (one color on the last layer), then permuting them (solving the ring around the last layer). The basic method has 77 algorithms (without the inverse of them), and is recognized as one of the fastest methods currently in use. [1] (http://www.ws.binghamton.edu/fridrich/cube.html)

F2L Alternatives: Methods that follow the same principle as Fridrich's method, but using different algorithms. Many of the algorithms are shared but there are a few differences, so there should be one to suit your fingers: Bob Burton: [2] (http://www.cubewhiz.com/) Shotaro 'Macky' Makisumi: [3] (http://cubefreak.hp.infoseek.co.jp/) Speedcubing.com Collection: speedcubinglovers.com

ZB method: This method was developed independently by Ron van Bruchem and Zbigniew Zborowski in 2003. After solving the cross and three c/e pairs, the final F2L pair is solved while orienting LL edges. This is known as ZBF2L. The ultimate method requires several hundred algorithms. To date, no one has learned the entire method. Lars Vandenbergh's site has ZBF2L algorithms, used in his VH system. [4] (http://www.cubezone.be/zbf2l.html) ZBLL algorithms can be found on Doug Li's webpage. [5]
VH method: Created by Lars Vandenbergh and Dan Harris, as a stepping stone from Fridrich to ZB. First, F2L without one c/e-pair is solved with Fridrich or some other method. Then the last pair is paired up, but not inserted. Then it's inserted to F2L and LL edges are oriented in one go. Then, using COLL, corners of LL are solved while preserving edge orientation. Then edges are permuted. [6] (http://www.cubestation.co.uk/cs2/)

Block methods

Petrus System: Created by Lars Petrus. One of the shortest methods in terms of face turns per solve, the Petrus method is often used in fewest moves contests. Petrus reasoned that as you construct layers, further organization of the cube's remaining pieces is restricted by what you have already done. In order for a layer-based solution to continue after the first layer had been constructed, the solved portion of the cube would have to be temporarily disassembled while the desired moves were made, then reassembled afterward. Petrus sought to get around this quagmire by solving the cube outwards from one corner, leaving him with unrestricted movement on several sides of the cube as he progressed. There are not as many algorithms to learn compared to the other F2L methods, but it takes a lot of dedication to master. The basis of the method is to create a 2x2x3 block on the cube, then proceed to solve a 3x3x2 block, but also flipping the edges on the Last Layer. Then the Last Layer is solved in two steps, first corners and then edges. [7] (http://lar5.com/cube/)

Heise method: Created by Ryan Heise, this method doesn't require any algorithms. First, one inner square and three outer squares are built intuitively. Then they are placed correctly while orienting remaining edges. After that you create two c/e-pairs, and solve the remaining edges. The last 3 corners are solved using a commutator. [8] (http://www.ryanheise.com/cube/)

Gilles Roux Method: Another unique method, but works in blocks like the Petrus method. You start by solving a 1x2x3 block and then solve another 1x2x3 block on the other side of the cube. Next you solve the last 4 corners and finally the edges and centers. Has only 24 algorithms to learn. [9] (http://grrroux.free.fr/method/Intro.html)

Corners first methods

Waterman Method: Created by Mark Waterman. Advanced corners first method, with about 90 algorithms to learn. Solve a face on L, do the corners on R and then solve the edges. An extremely fast method. [10] (http://www.rubikscube.info)

Jelinek Method: Created by Josef Jelinek. This method is very similar to Waterman's. [11] (http://rubikscube.info/)

Other methods

Salvia Method: Created by David Salvia, similar to Friedrich except the first two layers are built much differently and the final steps are CLL ELL. This method was built around freedom of movement. This means that most solves take between 33 and 54 moves (counting any movement of a side or slice as one move.) In the F2L many cubists do "the cross" first, this method does it last. While many methods use OLL PLL, Salvia uses CLL ELL. [12] (http://www.speedcubing.com/DavidJSalvia.html)

ZZ method: This is Zbigniew Zborowski's method. First, DF and DB edges are solved and all 12 edges are oriented. Then F2L without one c/e-pair is solved using RUL. Then the last pair and LL edges are solved in one go. After that LL corners are solved with ZZLL.

Ofapel method: This method starts similarly to the method described in this article, but after solving the first layer, the last layer corners are solved, then the last layer edges, then the middle layer edges. It is similar to corners-first, but the entire first layer is completed without leaving one edge unsolved. [13] (http://ofapel.free.fr/)

Other solution pages
- Picture Based Solution Guide (http://www.rubikssolver.com) - Pictures show each turn of the cube.
- Rubik's Cube Tutorial For Beginners - Video Lessons (http://www.youtube.com/watch?v=rPaq2UMLWXQ)
  Detailed steps which are easy to understand and follow for new cubers
- Beginner Solution to the Rubik's Cube (http://peter.stillhq.com/jasmine/rubikscubesolution.html) by Jasmine Lee
- A simple visual method (http://vanderblonk.com/cube/yy/) - using animations
- How to Solve the Rubik's Cube (http://jeays.net/rubiks.htm) by Mark Jeays
- Rubik's Cube for Beginners (http://www.learn2cube.com) by Alan Chang
- Cubeland (http://cubeland.free.fr/) A simple solution described with animations. Tricks to optimize for speed. By Christophe.
- Solving the Cube - Rubik's official website (http://www.rubiks.com/lvl3/index_lvl3.cfm?lan=eng&lvl1=commun&lvl2=cbegam&lvl3=compet) An easy to follow, step-by-step video of Tyson Mao teaching you how to solve the Rubik's cube. A world record holding speed-cuber, Mao was hired to teach Will Smith how to solve the Rubik's cube for his role in "The Pursuit of Happyness".
- Solution Hints Booklet step animation (http://www.ganpuzzle.com/hints3x3.htm) - Interactively shows how the 9 steps in booklet are done with animation accompanied by detailed description.

Alternative methods

- Solving the Rubik's Cube for Speed (http://lar5.com/cube/) , a block method by Lars Petrus
- Ultimate solution to the Rubik's Cube (http://helm.lu/cube/PhilipMarshall/) An edges-first method by Philip Marshall, requiring the memorization of only 2 algorithms and requiring an average of only 65 moves to solve.

Solving Programs

- Wedrans Cube Solution (http://www.wedran.com/cube) - Online solving program giving step by step instructions using a 3d animated cube.
- Automatic cube solver (http://wrongway.org/cube/solve.html)
- Cube solver with 3D Java applet (http://www.roobik.com/cgi-bin/rubix/rubix.cgi)
- Cube Explorer 4.10 (http://kociemba.org/cube.htm) - A fast program for finding optimal or near optimal solutions to the cube (less than 20 moves total!)

Background on math

- Wikipedia article: Optimal solutions for Rubik's Cube

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